

MATLAB 1.1

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% This script allows the user to input two vectors and
% then compute their dot product, cross product, sum,
% and difference
clear
vA = input('Enter vector A in the format [x y z]... \n > ');
if isempty(vA); vA = [0 0 0]; end % if the input is
                                % entered incorrectly set the vector to 0
vB = input('Enter vector B in the format [x y z]... \n > ');
if isempty(vB); vB = [0 0 0]; end
disp('Magnitude of A:')
disp(norm(vA)) % norm finds the magnitude of a
               % multi-dimensional vector
disp('Magnitude of B:')
disp(norm(vB))
disp('Unit vector in direction of A:')
disp(vA/norm(vA)) % unit vector is the vector
                  % divided by its magnitude
disp('Unit vector in direction of B:')
disp(vB/norm(vB))
disp('Sum A+B:')
disp(vA+vB)
disp('Difference A-B:')
disp(vA-vB)
disp('Dot product (A . B):')
disp(dot(vA,vB)) % dot takes the dot product of vectors
disp('Cross product (A x B):')
disp(cross(vA,vB)) % cross takes cross product of vectors

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SUMMARY

1. A field is a function that specifies a quantity in space. For example, $A(x, y, z)$ is a vector field, whereas $V(x, y, z)$ is a scalar field.
2. A vector A is uniquely specified by its magnitude and a unit vector along it, that is, $A = Aa_A$.
3. Multiplying two vectors A and B results in either a scalar $A \cdot B = AB \cos \theta_{AB}$ or a vector $A \times B = AB \sin \theta_{AB} a_n$. Multiplying three vectors A , B , and C yields a scalar $A \cdot (B \times C)$ or a vector $A \times (B \times C)$.
4. The scalar projection (or component) of vector A onto B is $A_B = A \cdot a_B$, whereas vector projection of A onto B is $A_B = A_B a_B$.

REVIEW
QUESTIONS

- 1.1 Tell which of the following quantities is not a vector: (a) force, (b) momentum, (c) acceleration, (d) work, (e) weight.
- 1.2 Which of the following is not a scalar field?
 - (a) Displacement of a mosquito in space
 - (b) Light intensity in a drawing room
 - (c) Temperature distribution in your classroom
 - (d) Atmospheric pressure in a given region
 - (e) Humidity of a city

1.3 Of the rectangular coordinate systems shown in Figure 1.13, which are not right handed?

1.4 Which of these is correct?

- (a) $\mathbf{A} \times \mathbf{A} = |\mathbf{A}|^2$
 (b) $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} = \mathbf{0}$
 (c) $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \cdot \mathbf{A}$

- (d) $\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_z$
 (e) $\mathbf{a}_k = \mathbf{a}_x - \mathbf{a}_y$, where $\mathbf{a}_k$ is a unit vector

1.5 Which of the following identities is not valid?

- (a) $\mathbf{a}(\mathbf{b} + \mathbf{c}) = \mathbf{ab} + \mathbf{bc}$
 (b) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
 (c) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
 (d) $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = -\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})$
 (e) $\mathbf{a}_A \cdot \mathbf{a}_B = \cos \theta_{AB}$

1.6 Which of the following statements are meaningless?

- (a) $\mathbf{A} \cdot \mathbf{B} + 2\mathbf{A} = \mathbf{0}$
 (b) $\mathbf{A} \cdot \mathbf{B} + 5 = 2\mathbf{A}$
 (c) $\mathbf{A}(\mathbf{A} + \mathbf{B}) + 2 = \mathbf{0}$
 (d) $\mathbf{A} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B} = \mathbf{0}$

1.7 Let $\mathbf{F} = 2\mathbf{a}_x - 6\mathbf{a}_y + 10\mathbf{a}_z$ and $\mathbf{G} = \mathbf{a}_x + G_y\mathbf{a}_y + 5\mathbf{a}_z$. If $\mathbf{F}$ and $\mathbf{G}$ have the same unit vector, G_y is

- (a) 6
 (b) -3
 (c) 0
 (d) 6

1.8 Given that $\mathbf{A} = \mathbf{a}_x + \alpha\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = \alpha\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, if $\mathbf{A}$ and $\mathbf{B}$ are normal to each other, α is

- (a) -2
 (b) -1/2
 (c) 0
 (d) 1
 (e) 2

1.9 The component of $6\mathbf{a}_x + 2\mathbf{a}_y - 3\mathbf{a}_z$ along $3\mathbf{a}_x - 4\mathbf{a}_y$ is

- (a) $-12\mathbf{a}_x - 9\mathbf{a}_y - 3\mathbf{a}_z$
 (b) $30\mathbf{a}_x - 40\mathbf{a}_y$
 (c) 10/7
 (d) 2
 (e) 10

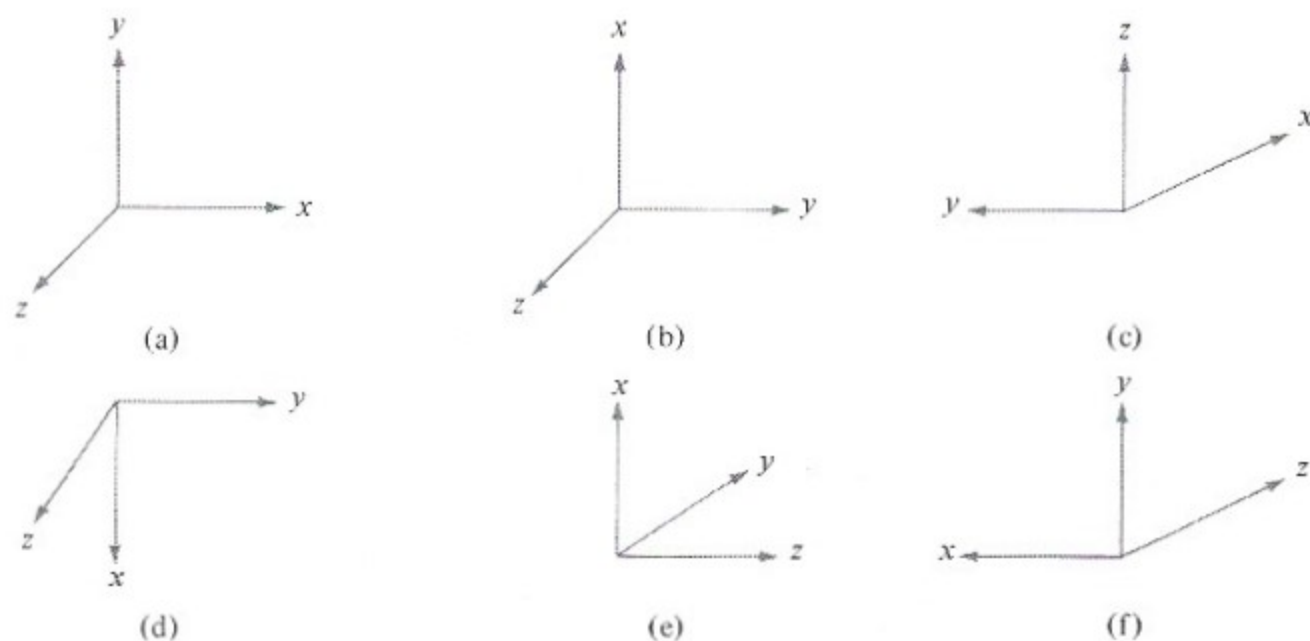


FIGURE 1.13 For Review Question 1.3.

1.10 Given $\mathbf{A} = -6\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$, the projection of $\mathbf{A}$ along $\mathbf{a}_y$ is

- (a) -12 (d) 7
(b) -4 (e) 12
(c) 3

Answers: 1.1d, 1.2a, 1.3b,e, 1.4b, 1.5a, 1.6a,b,c, 1.7b, 1.8b, 1.9d, 1.10c.

PROBLEMS

Section 1.4—Unit Vector

- 1.1 Determine the unit vector along the direction OP , where O is the origin and P is point $(4, -5, 1)$.
1.2 Find the unit vector along the line joining point $(2, 4, 4)$ to point $(-3, 2, 2)$.

Sections 1.5–1.7—Vector Addition, Subtraction, and Multiplication

1.3 If $\mathbf{A} = 4\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 12\mathbf{a}_x + 18\mathbf{a}_y - 8\mathbf{a}_z$, determine:

- (a) $\mathbf{A} - 3\mathbf{B}$
(b) $(2\mathbf{A} + 5\mathbf{B})/|\mathbf{B}|$
(c) $\mathbf{a}_x \times \mathbf{A}$
(d) $(\mathbf{B} \times \mathbf{a}_x) \cdot \mathbf{a}_y$

1.4 Given vectors $\mathbf{A} = 4\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -\mathbf{a}_x + 8\mathbf{a}_y + 5\mathbf{a}_z$, find (a) $\mathbf{A} - 2\mathbf{B}$, (b) $\mathbf{A} \cdot \mathbf{B}$, (c) $\mathbf{A} \times \mathbf{B}$.

1.5 Let $\mathbf{A} = 4\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, $\mathbf{B} = 3\mathbf{a}_x + 5\mathbf{a}_y + \mathbf{a}_z$, and $\mathbf{C} = \mathbf{a}_y - 7\mathbf{a}_z$. Find $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$.

1.6 Let $\mathbf{A} = \mathbf{a}_x - \mathbf{a}_z$, $\mathbf{B} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, $\mathbf{C} = \mathbf{a}_y + 2\mathbf{a}_z$, find:

- (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$
(b) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$
(c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$
(d) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$

1.7 If the position vectors of points T and S are $3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ and $4\mathbf{a}_x + 6\mathbf{a}_y + 2\mathbf{a}_z$, respectively, find (a) coordinates of T and S , (b) the distance vector from T to S , (c) the distance between T and S .

1.8 Let $\mathbf{A} = \alpha\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z$ and $\mathbf{B} = 4\mathbf{a}_x + \beta\mathbf{a}_y + 8\mathbf{a}_z$.

- (a) Find α and β if $\mathbf{A}$ and $\mathbf{B}$ are parallel.
(b) Determine the relationship between α and β if $\mathbf{B}$ is perpendicular to $\mathbf{A}$.

1.9 (a) Show that

$$(\mathbf{A} \cdot \mathbf{B})^2 + |\mathbf{A} \times \mathbf{B}|^2 = (AB)^2$$

(b) Show that

$$\mathbf{a}_x = \frac{\mathbf{a}_y \times \mathbf{a}_z}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \quad \mathbf{a}_y = \frac{\mathbf{a}_z \times \mathbf{a}_x}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \quad \mathbf{a}_z = \frac{\mathbf{a}_x \times \mathbf{a}_y}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}$$

1.10 Given that

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z$$

$$\mathbf{Q} = 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$

$$\mathbf{R} = -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

find: (a) $|\mathbf{P} + \mathbf{Q} - \mathbf{R}|$, (b) $\mathbf{P} \cdot \mathbf{Q} \times \mathbf{R}$, (c) $\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R}$, (d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R})$,
 (e) $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$, (f) $\cos \theta_{PR}$, (g) $\sin \theta_{PQ}$.

1.11 If $\mathbf{A} = 4\mathbf{a}_x - 6\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + 5\mathbf{a}_z$, find:

(a) $\mathbf{A} \cdot \mathbf{B} + 2|\mathbf{B}|^2$

(b) a unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$

1.12 Determine the dot product, cross product, and angle between

$$\mathbf{P} = 2\mathbf{a}_x - 6\mathbf{a}_y + 5\mathbf{a}_z \quad \text{and} \quad \mathbf{Q} = 3\mathbf{a}_y + \mathbf{a}_z$$

1.13 Show that vectors $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -2\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z$ are parallel.

1.14 Simplify the following expressions:

(a) $\mathbf{A} \times (\mathbf{A} \times \mathbf{B})$

(b) $\mathbf{A} \times [\mathbf{A} \times (\mathbf{A} \times \mathbf{B})]$

1.15 Find the area of the parallelogram formed by the vectors $\mathbf{D} = 4\mathbf{a}_x + \mathbf{a}_y + 5\mathbf{a}_z$ and $\mathbf{E} = -\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$.1.16 A right angle triangle has its corners located at $P_1(5, -3, 1)$, $P_2(1, -2, 4)$, and $P_3(3, 3, 5)$.
 (a) Which corner is a right angle? (b) Calculate the area of the triangle.1.17 Points P , Q , and R are located at $(-1, 4, 8)$, $(2, -1, 3)$, and $(-1, 2, 3)$, respectively. Determine (a) the distance between P and Q , (b) the distance vector from P to R , (c) the angle between QP and QR , (d) the area of triangle PQR , (e) the perimeter of triangle PQR .1.18 Two points $P(2, 4, -1)$ and $Q(12, 16, 9)$ form a straight line. Calculate the time taken for a sonar signal traveling at 300 m/s to get from the origin to the midpoint of PQ .1.19 Show that the dot and cross in the triple scalar product may be interchanged, that is, $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.*1.20 (a) Prove that $\mathbf{P} = \cos \theta_1 \mathbf{a}_x + \sin \theta_1 \mathbf{a}_y$ and $\mathbf{Q} = \cos \theta_2 \mathbf{a}_x + \sin \theta_2 \mathbf{a}_y$ are unit vectors in the xy -plane, respectively, making angles θ_1 and θ_2 with the x -axis.

(b) By means of dot product, obtain the formula for $\cos(\theta_2 - \theta_1)$. By similarly formulating $\mathbf{P}$ and $\mathbf{Q}$, obtain the formula for $\cos(\theta_2 + \theta_1)$.

(c) If θ is the angle between $\mathbf{P}$ and $\mathbf{Q}$, find $\frac{1}{2}|\mathbf{P} - \mathbf{Q}|$ in terms of θ .

1.21 Consider a rigid body rotating with a constant angular velocity ω radians per second about a fixed axis through O as in Figure 1.14. Let $\mathbf{r}$ be the distance vector from O to P , the position of a particle in the body. The magnitude of the velocity $\mathbf{u}$ of the body at P is $|\mathbf{u}| = d|\omega| = |\mathbf{r}| \sin \theta |\omega|$ or $\mathbf{u} = \omega \times \mathbf{r}$. If the rigid body is rotating at 3 rad/s about an axis parallel to $\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$ and passing through point $(2, -3, 1)$, determine the velocity of the body at $(1, 3, 4)$.

*Single asterisks indicate problems of intermediate difficulty.

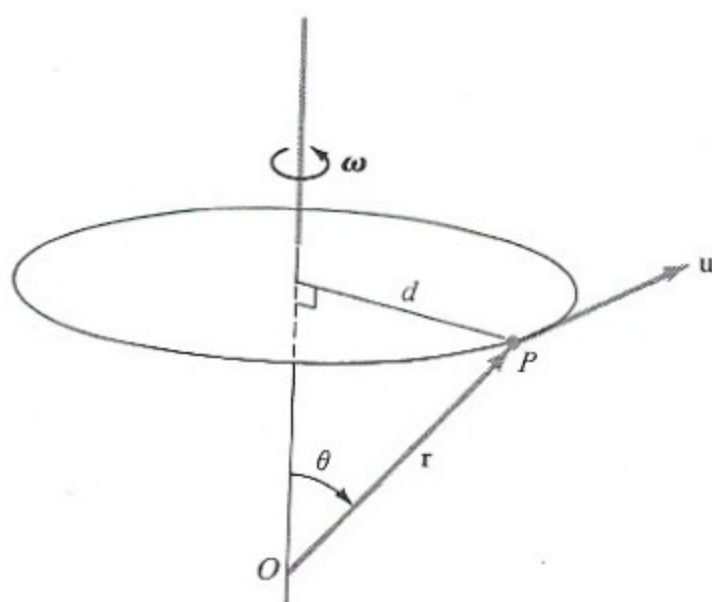


FIGURE 1.14 For Problem 1.21.

- 1.22 A cube of side 1 m has one corner placed at the origin. Determine the angle between the diagonals of the cube.
- 1.23 Given vectors $\mathbf{T} = 2\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{S} = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, find (a) the scalar projection of $\mathbf{T}$ on $\mathbf{S}$, (b) the vector projection of $\mathbf{S}$ on $\mathbf{T}$, (c) the smaller angle between $\mathbf{T}$ and $\mathbf{S}$.

Section 1.8—Components of a Vector

- 1.24 Given two vectors $\mathbf{A}$ and $\mathbf{B}$, show that the vector component of $\mathbf{A}$ perpendicular to $\mathbf{B}$ is
- $$\mathbf{C} = \mathbf{A} - \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}} \mathbf{B}$$
- 1.25 If $\mathbf{H} = 2xy\mathbf{a}_x - (x + z)\mathbf{a}_y + z^2\mathbf{a}_z$, find:
- A unit vector parallel to $\mathbf{H}$ at $P(1, 3, -2)$
 - The equation of the surface on which $|\mathbf{H}| = 10$
- 1.26 Given three vectors
- $$\begin{aligned}\mathbf{A} &= 4\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z \\ \mathbf{B} &= \mathbf{a}_x - \mathbf{a}_y \\ \mathbf{C} &= \mathbf{A} + \mathbf{B}\end{aligned}$$
- Find: (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$, (b) the vector component of $\mathbf{A}$ along $\mathbf{B}$.
- 1.27 Let $\mathbf{G} = x^2\mathbf{a}_x - y\mathbf{a}_y + 2z\mathbf{a}_z$ and $\mathbf{H} = yz\mathbf{a}_x + 3\mathbf{a}_y - xz\mathbf{a}_z$. At point $(1, -2, 3)$, (a) calculate the magnitude of $\mathbf{G}$ and $\mathbf{H}$, (b) determine $\mathbf{G} \cdot \mathbf{H}$, (c) find the angle between $\mathbf{G}$ and $\mathbf{H}$.
- 1.28 Determine the scalar component of vector $\mathbf{H} = y\mathbf{a}_x - x\mathbf{a}_z$ at point $P(1, 0, 3)$ that is directed toward point $Q(-2, 1, 4)$.
- 1.29 Given two vector fields
- $$\mathbf{D} = yz\mathbf{a}_x + xz\mathbf{a}_y + xy\mathbf{a}_z \quad \text{and} \quad \mathbf{E} = 5xy\mathbf{a}_x + 6(x^2 + 3)\mathbf{a}_y + 8z^2\mathbf{a}_z$$
- Evaluate $\mathbf{C} = \mathbf{D} + \mathbf{E}$ at point $P(-1, 2, 4)$. (a) Find the angle $\mathbf{C}$ makes with the x -axis at P .
- 1.30 $\mathbf{E}$ and $\mathbf{F}$ are vector fields given by $\mathbf{E} = 2x\mathbf{a}_x + \mathbf{a}_y + yz\mathbf{a}_z$ and $\mathbf{F} = xy\mathbf{a}_x - y^2\mathbf{a}_y + xyz\mathbf{a}_z$. Determine:
- $|\mathbf{E}|$ at $(1, 2, 3)$
 - The component of $\mathbf{E}$ along $\mathbf{F}$ at $(1, 2, 3)$
 - A vector perpendicular to both $\mathbf{E}$ and $\mathbf{F}$ at $(0, 1, -3)$ whose magnitude is unity